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GENERALISED REDUNDANTGIRTH DOMINATION NUMBER AND ITS CONNECTIVITY OF CORONA GRAPHS

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ABSTRACT

A set D⊂V($G \circ H$) is called a redundant girth dominating set of Corona graph G if every vertex in V-D is adjacent to at least one vertex in the girth(cycle) graph of G and N[x]-N[D-x]=∅. The minimum cardinality of a redundant girth dominating set of G is called redundant girth domination number of G denoted by $\gamma_{rg}(G \circ H)$.The connectivity $\kappa(G \circ H)$ of a Corona graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph.

KEYWORDS:

Redundant; *Girth,Domination Number;Connectivity; Redundant Girth domination number ;*

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1. INTRODUCTION

Consider a circle where each range of vertices of domain is changed from inner portion to a image or co-domain in the outer vertices . We call this circle as 'cipher circle'. In this way if we try to change the vertices. Now we use the same circle if we substitute for each outer vertices corresponding to the inner vertices. The range function we get it is a bijective mapping of relation which is called bijective range function of Corona Graph.For a given vertex v of a graph G , The open neighbourhood of v in G is the set $N_G(v)$ of all vertices of G that are adjacent to v. The degree $deg_G(v)$ of v refers to$|N_G(v)|$, and $\Delta(G) = \max\{deg_G(v)\colon v \epsilon V(G)\}$. The closed neighbourhood of v is the set $N_G[v]$ = $N_G(v)$ ∪ v for S ⊆ $V(G)$, $N_G(S)$ =$\bigcup_{v\epsilon s} N_G(v)$ and $N_G[v]$= $N_G(S)$∪ S. If $N_G[v]$==V(G), then S is a dominating set in G. The minimum cardinality among dominating sets in G is called the *domination number* of G and is denoted by $\gamma(G)$.

Definition :If T is a regular of degree 2,every component is a cycle and regular graphs of degree 3 are called cubic.

Definition :If all the edges of the girth are the edges of any other cycles in a graph G.

Theorem :Let x be a line of a connected graph G, The following statements are equivalent(1)x is a bridge of G.(2) x is not on any cycle of G.(3)There exist points u and v of G s.t the line x is on every path joining u and v.(4)There exists a partition of v into subsets U and W s.t for any points u∈Uand w∈W the line x is on every path joining u and w.

Theorem:Let G be a connected graph with atleast three points. The following statements are equivalent. (1)G is a block (2)Every two points of G lie on a common cycle (3)Every point and line of G lie on a common cycle (4) Every two lines of G lie on a common cycle (5)Given two points and one line of G, there is a path joining the points which contains the line (6)For every three distinct points of G, There is a path joining any two of them which contains the third.

A set$x \in S$ is said to be redundant in S if $N[x] \subseteq N[S-\{x\}]$ or N[x]-N[S-x]= $\emptyset$othewise x is said to be irredundant in S . Finally , S is called an irredundant set if all $x \in S$ are irredundant in S , Otherwise S is a redundant set .

The Corona G ∘ H of a graphs G and H is the graph obtained by taking one copy of G and $|V(G)|$ copies of H and then joining the i^{th} vertex of G to every vertex in the i^{th} copy of H . It is customary to denote by H_v that copy of H whose vertices are adjoined with the vertex v of G. In effect $G \circ H$ is composed of the subgraphs H_v+v joined together by the edges of G and its cartesian product of G and H. Moreover $V(G \circ H) = \bigcup_{v \epsilon V(G)} V(H_v + v)$ and let $S \subseteq V(G \circ H)$. If S is a dominating set in $G \circ H$ then $S \cap V(H_v + v) \neq \emptyset$, $\forall v \epsilon V(G)$

2.MAIN RESULTS:

Definition:2.0: A set D⊂V($G \circ H$) is called a redundant girth dominating set of Corona graph G if every vertex in V-D is adjacent to at least one vertex in the girth(cycle) graph of G and N[x]-N[D-x]=∅. The minimum cardinality of a redundant girth dominating set of G is called redundant girth domination number of G denoted by $\gamma_{rg}(G \circ H)$.The connectivity $\kappa(G \circ H)$ of a Corona graph G is the minimum number of vertices whose removal results in a disconnected or trivial graph.

Example 2.1: For any Corona graph , n is the number of vertices in G and m is the number of vertices in H for $|G|$=K_4 =$C_3 + v = 4$ and $\bigcup N(v_i) = C_3$ has girth dominating set of G and N[x]-N[D-x]=∅ with $\gamma_g(G)$=n-1=3 for n= 4 if Max{d(u_i, u_j)} =n-2,$i \neq j$ where $u_i \in C_3$ also κ(G)=n-1 for any complete graph of K_n which gives $\kappa(G \circ H)$=m . Hence $\gamma_{rg}(G \circ H)$+ $\kappa(G \circ H)$=(n-1)+(m)=n+m-1.

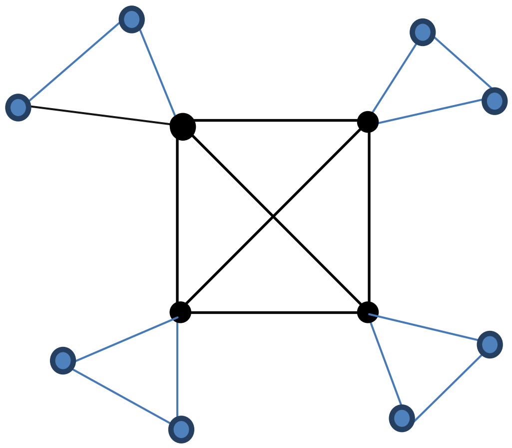

Figure 1: $|G|$=K_4 =$C_3 + v = 4$ and $\gamma_{rg}(G \circ H)$+ $\kappa(G \circ H)$=(n-1)+(m)=n+m-1.

Result 2.2: For any Corona graph, $|G|$=K_n =$C_{n-i} + iv = (n - i + i) = n$, i≥ 1 and $\bigcup N(v_i) = C_3$ also $|N(u_i) \cap N(V - D)| = 1$,i≠1 and N[x]-N[D-x]=∅ is a redundant girth dominating set of G with $\gamma_{rg}(G \circ H)$=n for n≥ 4 if Max{d(u_i, u_j)} ≥n-(i+1),$i \neq j$ where $u_i \in C_4$ also $\kappa(G \circ H)$=m for any complete graph of K_n.

Also the corona Graph of result 2.2.have $\gamma_{rg}(G \circ H)+\kappa(G \circ H)$= (n) +(m)=m+n

Result 2.3: For any Corona graph, |G| $=K_n$ $=C_{n-2}+v=n, C_{n-2}=n-v$ and $\cup N(v_i)=C_{n-2}$ also $|N(u_i) \cap (V-S)|=1$,i≠1 and N[x]-N[D-x]=∅ is a redundant girth dominating set of G with $\gamma_{rg}(G)$ ≥n-2 for n≥ 5 for every $v_i \in V-S$. Since if Max{d(u_i, u_j)}$=2$ then we can have the redundant girth dominating set and its $\gamma_{rg}(G)$=3. If Max{d(u_i, u_j)} =3 then we can have $\gamma_{rg}(G)$=4,Similarly we can have the redundant girth dominating set with $\gamma_{rg}(G \circ H)$=k if Max{d(u_i, u_j)} =k-1 and $|\cup N(v_i)| \geq k$, for every $v_i \in V-S$.

Hence for the graph K_n,n≥5 , $\gamma_{rg}(G \circ H)$=n-(k-1) here n-(k-1)≥ 3 and its $\delta(G \circ H)$=n-1 for every corona of complete graph K_n, that is $\kappa(G \circ H)$=m. Hence we have ,$\gamma_{rg}(G \circ H)+\kappa(G \circ H)$=(n-(k-1))+=n-k+1+m=n-k +m+1 where k=3,4.....

Theorem 2.4: For any graph G≅ $K_n=C_{n-i}+iv=n$ has a redundant girth dominating set of G with $\gamma_{rg}(G \circ H)$=n-i=3 and its $\gamma_{rg}(G \circ H)+\kappa(G \circ H)$ ≥m+ n-i and for every n≥ 4,i=1,2.....,

Case(i) Suppose n=4

For the graph $K_n=C_{n-1}+v=n$, $C_{n-1}=n-v$ and $\cup N(v_i)=C_{n-1}$ that is $|\cup N(v_i)|=n$-1 also $|N(u_i) \cap (V-D)|=1$,i≠1 has a girth dominating set of G , N[x]-N[D-x]=∅ with $\gamma_{rg}(G \circ H)$=n-1=3 and its δ(G)=n-1 for every complete graph K_n, that is $\kappa(G \circ H)$=m, Hence , $\gamma_{rg}(G \circ H)+\kappa(G \circ H)$=(n-1)+(m)=m+n-1 also G is isomorphic to K_4.

Case(ii) Suppose n=5,we have $\gamma_{rg}(G \circ H)+\kappa(G \circ H)$=(n-1)+(m)=m+n-2 , also G is isomorphic to K_5..

Case(iii) Suppose n=6, we have $\gamma_{rg}(G \circ H)+\kappa(G \circ H)$=(n-1)+(m)=m+n-3 ,also G is isomorphic to K_6.

Case(iv) Suppose n=7, we have γ_{rg} (G ∘ H)+ κ(G ∘ H)=(n-1)+(m)=m+n-4 also G is isomorphic to K_7.

In general for the graph K_n=C_{n-i} $+ iv = n$,for every n≥ 4,i=1,2....., $C_{n-i} = n - iv$ and $\cup N(v_i) = C_{n-i}$ has a girth dominating set of G that is $|\cup N(v_i)| = n - i$ also $|N(u_i) \cap (V - D)| = 1$,i≠1 with γ_{rg} (G)=n-i≥3 since Max{d(u_i , u_j)} $= n - i - 1$ and its δ(G)=n-1 for every complete graph K_n, that is κ(G ∘ H)=m, Hence we have γ_{rg} (G ∘ H)+ κ(G ∘ H) ≥m+n-i .

Theorem 2.5: For any Corona graph, $|G|$=K_n =$C_{n-1} + v = n$,C_{n-1}=n-1 and $\cup N(v_i) = C_3$ has a girth dominating set of G with γ_{rg} (G)=n-1 and its γ_{rg} (G ∘ H)+ κ(G ∘ H)= 2n-i and for every n=5,2≤i≤ 5

Proof: Suppose for any graph $|G|$=K_n =$C_{n-1} + v = n$,C_{n-1}=n-1 and $\cup N(v_i) = C_3$ has a girth dominating set of G with γ_{rg} (G ∘ H)=n-1=3 for n= 4 if Max{d(u_i , u_j)} ≥n-2,$i \neq j$ where $u_i \in C_3$ also $|N(u_i) \cap (V - D)| = 1$,i≠1 , N[x]-N[D-x]=$\emptyset$ also δ(G ∘ H)=m for any complete graph of K_n and κ(G)=n-1. Hence γ_{rg} (G ∘ H)+ κ(G ∘ H)=(n-1)+m=m+n-1

Subcase(i) Suppose for any graph $|G|$=K_n =$C_{n-1} + v = n$,C_{n-1}=n-1 and $\cup N(v_i) = C_4$ has girth dominating set of G with γ_{rg} (G)=n-1=4 for n= 5

If Max{d(u_i , u_j)} =n-3,$i \neq j$ where $u_i \in C_3$ also $|N(u_i) \cap (V - D)| = 1$,i≠1 also δ(G)=n-1 for any complete graph of K_n and but Max{d(u_i , u_j)} ≠n-2,$i \neq j$ that is κ(G)= $n - 1$ since $|N(u_i) \cap (v)| = 1$, N[x]-N[D-x]=$\emptyset$. Hence we have δ(G)=n-1 that is κ(G)=(n-1), Here G isomorphic to K_5. Hence γ_{rg} (G ∘ H)+ κ(G ∘ H)=(n-2)+m

If Max{d(u_i , u_j)} =n-2,$i \neq j$ where $u_i \in C_4$ also atleast one $|N(u_i) \cap (V - D)| = \emptyset$,i≠1 also δ(G)=n-1 for any complete graph of K_n and but κ(G)≠n-1 since $|N(u_i) \cap (v_1)| = |\{v_1, u_2, u_4\} \cap \{v_1\}| = 1$. Hence we have δ(G)=n-2 that is κ(G)=(n-2) with G isomorphic to K_5-2e, Hence γ_{rg} (G ∘ H)+ κ(G ∘ H)=(n-2)+m

If Max{d(u_i , u_j)} =n-1,$i \neq j$ where $u_i \in C_5$ also $|N(u_i) \cap (V - D)| = \emptyset$,i≠1 also δ(G)=n-1 for any complete graph of K_n and but κ(G)=4 since $|N(u_i) \cap (v_1)| \neq 1$. Hence we have δ(G)=n-3 that is κ(G)=(n-3) with $|M| = 1$,Here G isomorphic to K_5-3e. Hence γ_{rg} (G ∘ H)+ κ(G ∘ H)=(n-3)+m

Subcase(ii) If δ(G)=n-4

We have either Max{d(u_i, u_j)} =n-3,$i \neq j$ where $u_i \in C_3$also $|N(u_i) \cap (V-D)| = \emptyset$,i≠1 which implies that γ_{rg} (G)=n-2 also δ(G)=n-1 for any complete graph of K_n and but given κ(G)=n-4 since $|N(u_i) \cap (v_1)| = |\{v_1, u_2, u_4\} \cap \{v_1\}| = 1$. Hence we have δ(G)=n-4 that is κ(G)=(n-4) with G is isomorphic to K_5-4e, Hence γ_{rg} (G $\circ$ H)+ κ(G $\circ$ H)=(n-2)+m

Suppose ,If δ(G)=n-2 again since $|N(u_i) \cap (v_1)| = 1$ then for any graph $|G|$=K_n =$[C_{n-2} + 2v] - 2e = n$,C_{n-2}=n-2 and $\cup N(v_i) = C_3$ has a girth dominating set of G with γ_{rg} (G)=n-2=3 for n= 5 if Max{d(u_i, u_j)} =n-3,$i \neq j$ where $u_i \in C_3$ also $|N(u_i) \cap (V-S)| = 1$,i≠1,$|N(u_i) \cap (V-D)| = \emptyset$ also δ(G)=n-1 for any complete graph of K_n and κ(G)=n-2 . Hence γ_{rg} (G $\circ$ H)+ κ(G $\circ$ H)=(n-2)+m

Theorem 2.9: If $\cup N(v_i) = \cup_{i=1}^{3} u_i$ and $|\cup N(v_i)| \geq 3$ then $\cup u_i$ are the redundant girth dominating set of any Corona graph G and its γ_{rg} (G $\circ$ H)+ κ(G $\circ$ H)=n-rt+r, t≥k+1 and r≥1.

Proof: Given G be any Corona graph, we have in $|G| = |C_{n-k} + \cup_{i=1}^{k} v_i|$=n then S=$C_{n-k} = n - \cup_{i=1}^{k} v_i$;n≥ $k+3$ and S=n-k, S≥ $k+3-k$, S≥ 3 and if $\cup_{i=1}^{k} v_i$ is non adjacent with any vertices ofC_{n-k} then we have if $|N(u_i) \cap (V-D)| \neq n-k$, also $|N(u_i) \cap (V-D)| = \emptyset$ .Hence N(v_i) must have adjacent with $C_{n-k}$by atleast one vertex which gives $|N(u_i) \cap (V-S)| = 1$ and already we have S≥ 3, There fore we have $\cup N(v_i) = \cup_{i=1}^{3} u_i$ and $|\cup N(v_i)| \geq 3$ and in general we have $\cup N(v_i) = (u_1, u_2, \ldots\ldots u_{n-i})$, $|\cup N(v)| = n-i$ where $u_i \in C_{n-i}$, i=1,2,.... and $v_i \in V-D$. Hence γ_{rg} (G) ≥n-i and Max{d(u_i, u_j)} ≥n-(i+1),$i \neq j$.

Suppose G=$C_{n-k} + 2\cup_{i=1}^{k} v_i$ =n and then S=$C_{n-k} = n - 2\cup_{i=1}^{k} v_i$;n≥ $k+3$ and S=n-k, S≥ $k+3-k$, S≥ 3 and if 2 $\cup_{i=1}^{k} v_i$ is non adjacent with any vertices ofC_{n-k} then we have if$|N(u_i) \cap (V-S)|$ =1 or 2 .Hence N(v_i) must have adjacent with $C_{n-k}$by atleast one vertex which gives if $|N(u_i) \cap (V-D)| = 1$ also $|N(u_i) \cap (V-$ □)=∅ and □(G)=2k-1 and Max{d(□□,□□)}=□−□−1 .Hence □□□(G∘ □)+ κ(G $\circ$ H)=n-2i+m-1

If$|\mathcal{N}(u_i) \cap (V-D)|$=2 .Hence N($v_i$) must have adjacent with $C_{n-k}$by atleast two vertex which gives if $|\mathcal{N}(u_i) \cap (V-D)| = 2$ and δ(G)=2 and Max{d(u_i, u_j)} $= n-k-1$ .Hence γ_{rg} (G ∘ H)+ κ(G ∘ H)=n-2i+m-2.

Subcase: Suppose G=$C_{n-k} + 3\bigcup_{i=1}^{k} v_i$ =n then S=$C_{n-k} = n - 3\bigcup_{i=1}^{k} v_i$;n≥ $k+3$ and S=n-k, S≥ $k+3-k$, S≥ 3 and if 3 $\bigcup_{i=1}^{k} v_i$ is non adjacent with any vertices ofC_{n-k} then we have if$|\mathcal{N}(u_i) \cap (V-D)|$ =1 or 2 or 3 .Hence N(v_i) must have adjacent with $C_{n-k}$by atleast three vertex which gives if $|\mathcal{N}(u_i) \cap (V-S)| = 1$ or 2 or 3 , $|\mathcal{N}(u_i) \cap (V-D)|$ =∅ and δ(G)= 1 or 2 or 3 and Max{d(u_i, u_j)} $= n-k-1$ .Hence γ_{rg} (G ∘ H)+ κ(G ∘ H)=n-3i+m-1 or n-3i+m-2 or n-3i +m-3.

Subcase(i) Suppose G=$C_{n-k} + r\bigcup_{i=1}^{k} v_i$ =n then S=$C_{n-k} = n - r\bigcup_{i=1}^{k} v_i$;n≥ $k+3$ and S=n-k, S≥ $k+3-k$, S≥ 3 and if r $\bigcup_{i=1}^{k} v_i$ is non adjacent with any vertices ofC_{n-k} then we have if$|\mathcal{N}(u_i) \cap (V-D)|$ =1 or 2 or r. Hence N(v_i) must have adjacent with $C_{n-k}$by atleast r vertex which gives if $|\mathcal{N}(u_i) \cap (V-S)| = 1$or 2 or 3 or r , $|\mathcal{N}(u_i) \cap (V-D)|$ =∅ and δ(G)= 1 or 2 or 3 or r and Max{d(u_i, u_j)} $= n-k-1$ .Hence γ_{rg} (G ∘ H)+ κ(G ∘ H)=n-ri+m-1 or n-ri+m-2 or n-ri +m-3 or n-ri +s , s≥m-1.

Suppose put k=k+1 ,G=$C_{n-(k+1)} + r\bigcup_{i=k+1}^{t} v_i$ =n, t≥ $k+1$, that is n-(k+1)+t=n, S=$C_{n-(k+1)} = n - r\bigcup_{i=k+1}^{t} v_i$;n≥ $k+3$ and S=n-k, S≥ $k+3-k$, S≥ 3 and if r $\bigcup_{i=k+1}^{t} v_i$ is non adjacent with any vertices of$C_{n-(k+1)}$ then we have if $|\mathcal{N}(u_i) \cap (V-D)|$ =1 or 2 or r , $|N(u_i) \cap (V-D)|$ =∅. Hence N(v_i) must have adjacent with $C_{n-(k+1)}$by atleast r vertex which gives if $|\mathcal{N}(u_i) \cap (V-D)| = 1$or 2 or 3 or r and δ(G)= n-1 and Max{d(u_i, u_j)} $= n-k-1$. Hence γ_{rg} (G ∘ H)+ κ(G ∘ H)=n-r(k+1)+m-1, k≥1.

Suppose G=$C_{n-(k+1)} + r\bigcup_{i=k+1}^{t} v_i$ =n, t ≥ k , that is n-(k+1)+t=n, S=$C_{n-(k+1)} = n - r\bigcup_{i=k+1}^{t} v_i$;n≥ $t+3$ and S=n-t=3, S≥ $t+3-t$, S≥ 3 and if r $\bigcup_{t=k+1} v_i$ is non adjacent with any vertices of$C_{n-t} = n - rt$ then we have if $|\mathcal{N}(u_i) \cap (V-S)|$ =1 or 2 or r, |N(u_i)∩(V-D)|=∅. Hence N(v_i) must have adjacent with $C_{n-(k+1)}$by atleast r vertex which gives if $|\mathcal{N}(u_i) \cap (V-S)| = 1$or 2 or 3 or r and δ(G)= r and Max{d(u_i , u_j)} $= n - (k + 1) - 1$ =n-k-1-1=n-k-2 . Hence γ_{rg} (G $\circ$ H)+ κ(G $\circ$ H)=n-rt+r+m.

3.CONCLUSION :

In this paper we found an upper bound for the Redundant girth domination number and its connectivity of Corona graph and the Relationships between Redundant girth domination numbers and characterized the corresponding extremal Corona graphs.

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